

FINAL TERM EXAMINATION
Fall 2008
(Session - 1)

Calculus & Analytical Geometry-I

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UPDATED VERSION

Exclusive thanks to Mahar Azahar (Lodhran)

Question No: 1 (Marks: 1) - Please choose one

_____ If $y = f(x)$ then the average rate of change of y with respect to x over the interval $[x_0, x_1]$ is the Joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ on the graph of f

- ▶ Slope of the secant line
- ▶ **Slope of tangent line**
- ▶ Secant line
- ▶ none of these

Question No: 2 (Marks: 1) - Please choose one

Natural domain of $\frac{(x^2 - 4)}{(x - 2)}$ is

- ▶ **$(-\infty, 2) \cup (2, +\infty)$**
- ▶ $(-\infty, 2)$
- ▶ $(-\infty, 0)$
- ▶ None of these

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Question No: 3 (Marks: 1) - Please choose one

The equation $(x+4)^2 + (y-1)^2 = 6$ represents a circle having center at and radius

- $(-4, 1), \sqrt{6}$
- $(-4, 1), 6$
- $(-4, -1), \sqrt{6}$
- None of these

Question No: 4 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that if $\rho > 1$ then the series

$\rho = \lim_{k \rightarrow \infty} \sqrt[k]{u_k} = \lim_{k \rightarrow \infty} (u_k)^{\frac{1}{k}}$

- Converges
- Diverges
- May converge or diverge
- None of these



Question No: 5 (Marks: 1) - Please choose one

series $\sum u_k$ and $\sum v_k$ are convergent series then will beand.....

$(\sum u_k + \sum v_k)$ and $(\sum u_k - \sum v_k)$

- Convergent, convergent
- Divergent, divergent
- Convergent, divergent
- Divergent, convergent

Question No: 6 (Marks: 1) - Please choose one

notation $\left\{\frac{1}{2^n}\right\}_{-1}^n$ represents the sequence

The

$2, 1, \frac{1}{2}, \frac{1}{4}, \dots$

0, 1, 2, 3...

$0, 1, \frac{1}{2}, \frac{1}{4}, \dots$

None of these

Question No: 7 (Marks: 1) - Please choose one

_____ If f is continuous on $(a, b]$ but does not have a limit from the right then the integral

$$\int_a^b f(x) dx = \lim_{l \rightarrow a} \int_l^b f(x) dx$$

defined by _____ is called Integral

Improper

Proper

None of these

Question No: 8 (Marks: 1) - Please choose one

_____ An object is displaced 1m by a force of 1N then the work done W is

2

0

None of these

1

Question No: 9 (Marks: 1) - Please choose one

_____ If f is a smooth function on $[a, b]$ then the arc length L of the curve $y=f(x)$ from $x=a$ to $x=b$ will be

$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$

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$$L = \int_a^b \sqrt{1 + [f'(x)]} dx$$



$$L = \int_0^a \sqrt{1 + [f'(x)]} dy$$



▶ None of these

Question No: 10 (Marks: 1) - Please choose one

_____ If f is a smooth function on $[0,3]$ then the arc length L of the curve $y=f(x)$ from $x=0$ to $x=3$ will be

$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dx$$



$$L = \int_a^b \sqrt{1 + [f'(x)]^2}$$



$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dy$$



▶ None of these

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Question No: 11 (Marks: 1) - Please choose one

_____ By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between $y = 3x$ and $y = 2x^2$ is revolved about the x -axis

$$V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2) dx$$



$$V = \int_0^{\frac{3}{2}} x(3x - 2x^2) dx$$



$$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2) dx$$



▶ None of these

Question No: 12 (Marks: 1) - Please choose one

By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis is represented by

$$V = \int_0^3 2\pi x(x - x^2) dx$$



$$V = \int_0^1 x(x - x^2) dx$$



$$V = \int_0^1 2\pi(x - x^2) dx$$



▶ None of these

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Question No: 13 (Marks: 1) - Please choose one

If

$$\int_a^a f(x) dx =$$

a is in the domain of f, then



$$f'(x)$$



$$f(x)$$

▶ 0

▶ None of these

Question No: 14 (Marks: 1) - Please choose one

$$\int_0^2 x^2 dx$$

Consider the integral $\int_0^2 x^2 dx$, the area on right is bounded by

▶ $y = x^2$

▶ $x = 2$

▶ $x = 0$

▶ None of these

Question No: 15 (Marks: 1) - Please choose one

The series $1 - 3 + 5 - 7 + 9 - 11$ may written as in sigma notation

$$\sum_{k=0}^{k=5} (-1)^k (2k + 1)$$

▶

$$\sum_{k=1}^{k=5} (-1)^k (2k + 1)$$

▶

$$\sum_{k=1}^{k=5} (2k + 1)$$

▶

▶ None of these

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Question No: 16 (Marks: 1) - Please choose one

$4^2 + 5^2 + 6^2 + 7^2$ in sigma notation may be represented as

$$\sum_{k=2}^{k=7} k^2$$



$$\sum_{k=2}^{k=7} (k+1)^2$$



$$\sum_{k=4}^{k=7} k^2$$

▶ None of these

Question No: 17 (Marks: 1) - Please choose one

_____ If
a function f is on a closed interval [a,b], then f has both a maximum and minimum value on [a,b]

▶ **Continuous**

▶ Discontinuous

▶ Differentiable

▶ None of these

Question No: 18 (Marks: 1) - Please choose one

_____ Let
f be a function on an interval, and x_1 and x_2 denote the points in that interval, if
 $f(x_1) < f(x_2)$
whenever
 $x_1 < x_2$
then the we can say that f is

▶ **Increasing function**

▶ **Decreasing function**

▶ Constant function

▶ None of these

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Question No: 19 (Marks: 1) - Please choose one

_____ If
a function satisfies the conditions
f(c) is defined

$$\lim_{x \rightarrow c^+} f(x)$$

Exists

$$\lim_{x \rightarrow c^+} f(x) = f(c)$$

Then the function is said to be

► **Continuous at c**

- Continuous from left at c
- Continuous from right at c
- None of these

Question No: 20 (Marks: 1) - Please choose one

For a function $f(x)$ to be continuous on interval (a,b) the function must be continuous

► **At all point in (a,b)**

- Only at a and b
- At mid point of a and b
- None of these



Question No: 21 (Marks: 2)

$$a_{n+1} = \frac{1}{3} \left(a_n + \frac{1}{a_n} \right) \text{ for } n \geq 1 \text{ and } a_1 = 2$$

Write down the first two term of the sequence

Question No: 22 (Marks: 2)

Find the integral of the surface area of the portion of the sphere generated by revolving the

$$y = \sqrt{2-x^2}, 0 \leq x \leq \frac{1}{3}$$

curve

(Note: Just find the integral do not solve the integral)

Question No: 23 (Marks: 2)

$$\int_2^5 f(x)dx - \int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$$

Calculate $\int_2^5 f(x)dx$ if

$$\int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$$

$$\int_2^5 f(x)dx = 7 + 2 - 5 = 4$$

Question No: 24 (Marks: 3)

the first two Taylor polynomials for $\ln x$ about $x = 3$

Find

Question No: 25 (Marks: 3)

the curve $y = x^{\frac{3}{2}}$; $0 \leq y \leq 2$, then find the surface area generated by revolving the curve. (But do not evaluate)

Let

Question No: 26 (Marks: 3)

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{7225}$$

Express the sum in sigma notation but do not evaluate.

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{7225}$$

$$\sum_{k=1}^{7225} k^3 + 1$$

Question No: 27 (Marks: 5)

the first four nonzero terms of the Taylor series generated by f at $x = a$

Find

$$f(x) = \frac{1}{1-x} \text{ at } x = 2$$

Question No: 28 (Marks: 5)

Evaluate the Definite Integral using the First fundamental Theorem of Calculus

$$\int_0^1 (x^5 - x^3 + 2x) dx$$
$$\text{Let } u = (x^5 - x^3 + 2x)$$
$$\int_0^1 (u) dx$$

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Question No: 29 (Marks: 5)

Express the definite integrals as limits (Do not evaluate the integrals)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos x) dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos x) dx$$
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \lim_{\max \Delta x \rightarrow 0} \sum_{i=1}^n (1 + \cos x) dx$$

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Question No: 30 (Marks: 10)

Find

the region enclosed by the curves and also find the area

$$y = x^2, y = \sqrt{x}, x = \frac{1}{4}, x = 1$$

Question No: 31 (Marks: 10)

Use x_k^* as the left end point of each subinterval to find the area under $y = mx$ over the interval $[a, b]$, where $m > 0$ and $a \geq 0$

Solution on next page

Suppose $a = 1$ $b = 2$ so $[a, b] = [1, 2]$
 $x_k^* = x_{k-1} = a + (k-1)\Delta x$ (formula for left end point)
 $\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$
Suppose k th has area
 $f(x_{k^*})\Delta x = x_{k^*}\Delta x$
 $\left[1 + \frac{k}{n}\right]\Delta x$
 $\left[1 + \frac{k}{n}\right]\frac{1}{n}$
 $\sum_{k=1}^n f(x_{k^*})\Delta x = \sum_{k=1}^n \left[1 + \frac{k}{n}\right]\frac{1}{n}$
Area by solving
 $A = \lim_{\delta x \rightarrow 0} \sum_{k=1}^n f(x_{k^*})\Delta x = \lim_{\delta x \rightarrow 0} \left[\frac{3}{2} - 1 + \frac{1}{2n} \right]$
 $= \frac{3}{2} - 1 + 0$

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.