FINALTERM EXAMINATION 2009
(Session - 1)

Calculus & Analytical Geometry-I

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UPDATED VERSION

Exclusive thanks to Mahar Azahar (Lodhran)

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative ………… At x_0

- ► Minima
- ► Maxima
- ► None of these

Question No: 2 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$ then f has relative ………… At x_0

- ► Minima
- ► Maxima
- ► None of these

Question No: 3 (Marks: 1) - Please choose one

$$\lim_{x\to 0}\frac{\sin 2x}{x} = -----$$

- ► 2
- ► 4

- ▶ 1
- ▶ ∞

Question No: 4 (Marks: 1) - Please choose one

$$\lim_{x\to 0^+}\frac{\ln x}{1/x} = -----$$

- ▶ 1
- ▶ 0
- ▶ e
- ▶ None of these

Question No: 5 (Marks: 1) - Please choose one

$$\frac{d(\tan\ x)}{dx} =$$

- ▶ $\sec x$
- ▶ $\sec^2 x$
- ▶ $co\sec x$
- ▶ $co\sec^2 x$

Question No: 6 (Marks: 1) - Please choose one

If $xy = 4$ then $\frac{dy}{dx} =$

- ▶ 0
- ▶ $\frac{-1}{x^2}$
- ▶ $\frac{4}{x^2}$
- ▶ $\frac{-4}{x^2}$

Question No: 7 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$\frac{d}{dx}\left((c)\,\{h(x)\}\right)$ = ________

- ▶ 0
- ▶ $\frac{d}{dx}\left(h(x)\right)$
- ▶ $\frac{d}{dx}\left(h(cx)\right)$
- ▶ $c\,\frac{d}{dx}\left(h(x)\right)$

Question No: 8 (Marks: 1) - Please choose one

Suppose that f and g are differentiable functions of x then

$\frac{d}{dx}[f][g] =$

- ▶ $\frac{[f'][g]-[f][g']}{g^2}$
- ▶ $[f'][g']$
- ▶ $[f'][g]+[f][g']$
- ▶ $[f'][g]-[f][g']$

Question No: 9 (Marks: 1) - Please choose one

The power rule, $\frac{d}{dx}[x^n] = nx^{n-1}$ holds if n is __________

- ▶ An integer
- ▶ A rational number
- ▶ An irrational number
- ▶ All of the above

Question No: 10 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denotes two distinct points in that interval. If $f(x_1) = f(x_2)$ for all points x_1 and x_2 then which of the following statement is correct?

- ▶ f is a decreasing function
- ▶ f is an increasing function
- ▶ f is a constant function

Question No: 11 (Marks: 1) - Please choose one

If $f''(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- ▶ f is concave up on (a, b).
- ▶ f is concave down on (a, b)
- ▶ f is linear on (a, b).

Question No: 12 (Marks: 1) - Please choose one

What does 'n' represent in Riemann Sum $\sum_{k=1}^{n} f(x^*_k)\Delta x_k$?

- ▶ No. of Circles
- ▶ No. of Rectangles
- ▶ No. of Loops
- ▶ No. of Squares

Question No: 13 (Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x \to -\infty} f(x) = +\infty \ and \ \lim_{x \to +\infty} f(x) = +\infty$ then f has __________ on $(-\infty, +\infty)$

- ▶ maximum value but no minimum
- ▶ minimum value but no maximum
- ▶ both maximum and minimum value

Question No: 14 (Marks: 1) - Please choose one

The expression $\int_2^t \frac{x^2}{2} dx$, represents a function of :

- ▶ t
- ▶ x
- ▶ 2
- ▶ Both t and x

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Question No: 15 (Marks: 1) - Please choose one

$\int cf(x)dx =$ ________

if c is a constant

- ▶ 0
- ▶ c
- ▶ $\int f(cx)dx$
- ▶ $c\int f(x)dx$

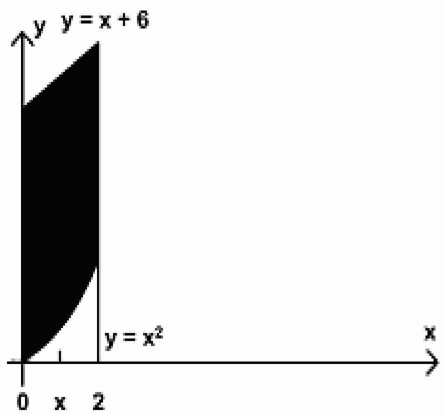
Question No: 16 (Marks: 1) - Please choose one

Sigma notation is represented by which of the following Greek letter?

- ▶ χ
- ▶ η
- ▶ Σ
- ▶ ψ

Question No: 17 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by **:**

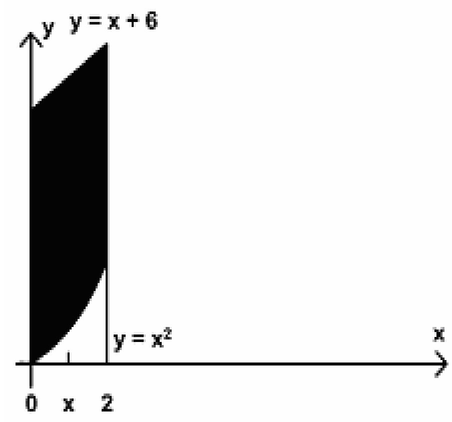


- ▶ $y = x + 6$
- ▶ $y = x^2$
- ▶ $x = 2$

▶ $x = 0$

Question No: 18 (Marks: 1) - Please choose one

In the following figure, the area bounded on the sides by the lines are **:**



▶ $x = 0$

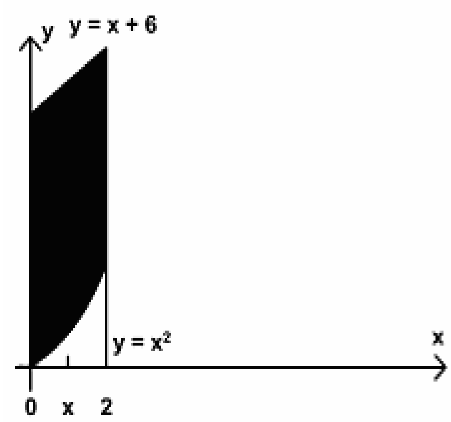
▶ $x = 2$

▶ $x = 0 \; and \; x = 2$

▶ $x = 6$

Question No: 19 (Marks: 1) - Please choose one

What is the area of the region in the following figure?



▶ $A = \int_0^2 \left[(x+6) - (x^2) \right] dx$

▶ $A = \int_x^2 \left[(x+6) - (x^2) \right] dx$

$$A = \int_0^2 \left[(x+6) + (x^2) \right] dx$$

▶

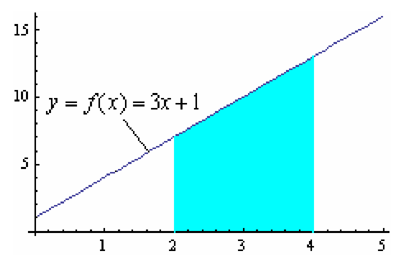
$$A = \int_0^x \left[(x+6) - (x^2) \right] dx$$

▶

Question No: 20 (Marks: 1) - Please choose one

Which of the following is approximate area under the curve $y = f(x) = 3x + 1$ over the interval $[2, 4]$, evaluated by using the formula

$$Area = f(x_1^*)\, \Delta x + f(x_2^*)\, \Delta x$$



If the interval $[2, 4]$ is divided into two sub-intervals of equal

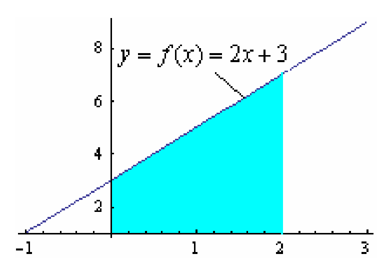
length and x_1^* and x_2^* are left endpoint of each sub-interval.

- ▶ 17
- ▶ 20
- ▶ 23

Question No: 21 (Marks: 1) - Please choose one

Which of the following is approximate area under the curve $y = f(x) = 2x + 3$ over the interval $[0, 2]$, evaluated by using the formula

$$Area = f(x_1^*)\, \Delta x + f(x_2^*)\, \Delta x$$



If the interval $[0, 2]$ is divided into two sub-intervals of equal length and x_1^* and x_2^* are right endpoint of each sub-interval.

- ▶ 8
- ▶ 10
- ▶ 12

Question No: 22 (Marks: 1) - Please choose one

$$If \;\; x > 0 \; then \; \frac{d}{dx}[\ln x] = \_\_\_\_\_\_\_\_\_\_$$

▶ 1

▶ x

▶ $\frac{1}{x}$

▶ $\ln\frac{1}{x}$

Question No: 23 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on [a,b] and c is a constant, then $\int_a^b c\,[f(x)+g(x)]\,dx =$ ______________

▶ $\int_a^b f(cx)dx+\int_a^b g(cx)dx$

▶ $\int_a^b f(x)\ dx+\int_a^b g(x)]\ dx$

▶ $c\int_a^b f(x)dx+c\int_a^b g(x)dx$

▶ 0

Question No: 24 (Marks: 1) - Please choose one

If the function f is continuous on [a,b] and if $f(x)\geq 0$ for all x in [a,b], then which of the following gives area under the curve $y=f(x)$ over the interval [a,b]?

▶ $\lim_{x\to\infty}\sum_{k=1}^{n}[x_k][f(x_k)] \qquad where\ n\ is\ number\ of\ subdivisions\ of\ [a,b]$

▶ $\int_a^b f(x)\ dx$

▶ $\pi[radius]^2$

▶ (Width) (Height)

Question No: 25 (Marks: 1) - Please choose one

Let region R in the first quadrant enclosed between $y = 3x \quad and \quad y = 2x^2$ is revolved about the
x-axis .Which of the following equation gives the volume of a solid by cylindrical shells?

► $V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2)dx$

► $V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$

► $V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$

► $V = \int_{-1}^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$

Question No: 26 (Marks: 1) - Please choose one

Let f is a smooth function on [a, b]. What will be the arc length L of the curve y = f(x) from x = a to x = b?

► $L = \int_b^a \sqrt{1 + [f'(x)]}dy$

► $L = \int_a^b \sqrt{1 + [f'(x)]^2}dx$

$$L = \int_0^a \sqrt{1+[f'(x)]}dy$$

▶

$$L = \int_a^b \sqrt{1+[f'(x)]}dx$$

▶

Question No: 27 (Marks: 1) - Please choose one

If f is continuous on (a, b] but does not have a limit from the right then the integral defined by $\int_a^b f(x)dx = \lim_{l \to a} \int_l^b f(x)dx$ is called **:**

- ▶ Improper
- ▶ Proper
- ▶ Line

Question No: 28 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as**:**

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} < 1$ then the sequence is known as**:**

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 30 (Marks: 1) - Please choose one

$$\int \frac{3x^2+4x+1}{x^3+2x^2+x-3}\,dx$$

Consider the indefinite integral

Let $t = x^3+2x^2+x-3$

Is the following substitution correct?

$$\int \frac{3x^2+4x+1}{x^3+2x^2+x-3}\,dx = \int \frac{1}{t}\,dt$$

- ► Yes
- ► No

Question No: 31 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$

if $\rho = 1$, then which of the following is true?

- ► Converges
- ► Diverges
- ► May converges or diverges
- ► Gives no information

Question No: 32 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that

$\rho = \lim_{k\to\infty} \sqrt[k]{u_k} = \lim_{k\to\infty} (u_k)^{\frac{1}{k}}$ if $\rho = 1$, then which of the following is true?

- ► Converges
- ► Diverges
- ► May converges or diverges
- ► Gives no information

Question No: 33 (Marks: 1) - Please choose one

If the series $\sum_{k=1}^{\infty} |u_k| = |u_1| + |u_2| + |u_3| + \ldots |u_k| + \ldots$ converges , then which of the following is true for $\sum_{k=1}^{\infty} u_k = u_1 + u_2 + u_2 + \ldots u_k + \ldots$?

- ▶ Converges
- ▶ Diverges
- ▶ Gives no information

Question No: 34 (Marks: 1) - Please choose one

Let $\sum u_k$ be a series with nonzero terms and suppose that $\rho = \lim_{k \to \infty} \frac{|u_{k+1}|}{|u_k|}$

if $\rho = +\infty$, then which of the following is true?

- ▶ Then the series $\sum u_k$ diverges
- ▶ The series $\sum u_k$ converges absolutely and therefore converges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 35 (Marks: 1) - Please choose one

$\int_{-1}^{1} (x-1)\, dx =$ ____________

- ▶ -2
- ▶ 0
- ▶ 2
- ▶ 4

Question No: 36 (Marks: 1) - Please choose one

How many critical points exist for a function f if

$f'(x) = (x-3)(x-2)$

- Zero
- One
- Two
- Four

Question No: 37 (Marks: 1) - Please choose one

$\log_b ac =$ ________

- $\log_b a + \log_b c$
- $\log_b a - \log_b c$
- $\dfrac{\log_b a}{\log_b c}$
- $(\log_b a)(\log_b c)$

Question No: 38 (Marks: 1) - Please choose one

$\log_b a^r =$ __________

- $a \log_b r$
- $r \log_b a$
- $\dfrac{\log_b a}{\log_b r}$
- $\log_b a + \log_b r$

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Question No: 39 (Marks: 1) - Please choose one

Let $y = \dfrac{2\sqrt{2}}{3} x^{\frac{3}{2}} \; ; \; 0 \le x \le 2$ then which of the following is the length of the curve?

$$L=\int_0^2 \sqrt{\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

▶

$$L=\int \sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

▶

$$L=\int_0^2 \sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

▶

$$L=\int_0^2 \sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]}\,dx$$

▶

Question No: 40 (Marks: 1) - Please choose one

Which of the following are *first two* terms for the Taylor series of $f(x)=e^{-x}$ at *x* = 0?

$1+(1)(x-0)$

▶

$1+(-1)(x+0)$

▶

$1+(-1)(x-0)$

▶

$(-1)(x-0)$

▶

Question No: 41 (Marks: 2)

$$\int_2^3 (1-x)dx$$

Evaluate the integral

$$\int_2^3 (1-x)dx$$
$$= | x - x^2 \frac{1}{2} |_2^3$$
$$= \frac{1}{2} | 2x - x^2 |_2^3$$
$$= \frac{1}{2}(2(3-2)-(3-2)^2)$$
$$= \frac{1}{2}(2-1)$$
$$= \frac{1}{2}$$

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Question No: 42 (Marks: 2)

$$\int_2^{+\infty} \frac{dx}{x^2}$$

Evaluate the improper integral

Question No: 43 (Marks: 2)

A function $f(x) = x^2 - 4x - 9$ has critical point 2 in an interval [0, 5]. Find the maximum value of the function and point having this value.

Question No: 44 (Marks: 3)

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Evaluate:

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Question No: 45 (Marks: 3)

Find the area of the region bounded by the curve $y = x^2 \ , \ x > 0$, and bounded on the sides by the lines $y = 1$ and $y = 4$

$y = x^2 \ , \ x > 0$

So we have

$$A=\int_{1}^{4} x^2 dx$$
$$=|\frac{x^3}{3}|_1^4$$
$$=\frac{1}{3}(4-1)^3$$
$$=\frac{1}{3}(3)^3$$
$$=9$$

Question No: 46 (Marks: 3)

Determine whether the following sequence converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty}\frac{5n^2-1}{20n+7n^2}$$

Question No: 47 (Marks: 5)

Use the Alternating series Test to determine whether the given series converges

$$\sum_{1}^{\infty}\frac{(-1)^{n-1}\ .\ n!}{2^n}$$

Question No: 48 (Marks: 5)

Evaluate the integral

$$\int_{\frac{\pi}{2}}^{0}\frac{1+\cos 2t}{2}dt$$

Solution

$$\int_{\frac{\pi}{2}}^{0} \frac{1+\cos 2t}{2} dt$$

$$u = 2t$$

$$\frac{du}{dt} = 2dt$$

$$du = 2dt$$

$$so$$

$$= \frac{1}{4} \int_{\frac{\pi}{2}}^{0} 1 + \cos u du$$

$$= \frac{1}{4} | u + \sin u |_{\frac{\pi}{2}}^{0}$$

$$= \frac{1}{4} | 2t + \sin 2t |_{\frac{\pi}{2}}^{0}$$

$$= \frac{1}{4} (2\frac{\pi}{2} + \sin 2\frac{\pi}{2})$$

$$= \frac{1}{4} (\pi + \sin \pi)$$

$$= \frac{1}{4} (\pi + 0)$$

$$= \frac{\pi}{4}$$

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Question No: 49 (Marks: 5)

Evaluate the sums

$$\sum_{k=1}^{5} k(3k+5)$$

$$= 1(3+5) + 2(6+5) + 3(9+5) + 4(12+5) + 5(15+5)$$

$$= 8 + 22 + 3(45) + 4(60) + 5(75)$$

$$= 8 + 22 + 135 + 240 + 375$$

$$= 780$$

Question No: 50 (Marks: 10)

Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x – axis.

$y = 1 + x^3$, $x = 1, x = 2, y = 0$

from $V = \int_a^b \pi [f(x)]^2 dx$

$V = \int_1^2 \pi [1 + x^3]^2 dx$

$V = \int_1^2 \pi [1 + x^5 + 2x^3] dx$

$V = \pi \int_1^2 (1 + x^5 + 2x^3) dx$

$V = \pi \mid (x + \frac{1}{6}x^6 + \frac{1}{2}x^4) \mid_1^2$

$V = \pi((2-1) + \frac{1}{6}(2-1)^6 + \frac{1}{2}(2-1)^4)$

$V = \pi\{((2-1) + \frac{1}{6}(2-1)^6 + \frac{1}{2}(2-1)^4)\}$

$V = \pi((1) + \frac{1}{6} + \frac{1}{2}))$

$V = \frac{\pi(6+1+3)}{6}$

$V = \frac{\pi(10)}{6} = \pi\frac{5}{3}$

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This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.