MTH401

Final Term Examination – Spring 2006

Time Allowed: 150 Minutes

Question No. 1 — **Marks : 5**

Determine the singular points of the differential equation and classify them as regular or irregular.

$$(x^2-9)^2 y'' + (x+3)y' + 2y =$$

Question No. 2 — **Marks : 10**

Solve the given Bernoulli equation

$$\frac{dy}{dx} + y = y^2$$

Question No. 3 — **Marks : 2**

Which of the following is singular point of the equation

$$2x^2y'' + 10xy' + (x-1)y = 0$$

- $x = 0$
- $x = 2$
- $x = 1$
- None of other

Question No. 4 — **Marks : 10**

Find the eigen value and eigen vector of the following system of linear differential equation

$$X' = \begin{pmatrix} 9 & -12 \\ 3 & -3 \end{pmatrix} X$$

Question No. 5 **Marks : 10**

Find solution of the differential equation

$$9y' + y =$$

0

[redacted]

Question No. 6 **Marks : 2**

The non-trivial solution of the system exists only when

$\det(A - \lambda I) = 0$

- True

[redacted]

Question No. 7 **Marks : 2**

The form of the particular solution for the differential equation $y' - y = x^2 e^{3x}$ is

$y_p = \quad {}^{x} + B_1 x + B_0$

Ae

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$y_p = (Ax + B)e^x$

[redacted]

$y_p = (Ax^2 + Bx + C)e^{3x}$

-
- None of above

Question No. 8 **Marks : 10**

Solve the following homogeneous system of differential equations

$$\frac{dx}{dt} - 7x + \frac{dy}{dt} = 3e^t$$

$$3\frac{dx}{dt} - 2x + \frac{dy}{dt} = 2e^t$$

[redacted]

(Just find the general solution of the equation).

Question No. 9 **Marks : 5**

Solve the differential equation

$y'' - 2y' + y = 0$

Question No. 10 **Marks : 2**

Which of the following is Legendre's Equation

- $$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 7y = 0$$
- $$(1 - x^2)\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 35y = 0$$
- $$\frac{d}{dx}\left[(1 - x^2)\frac{dy}{dx}\right] + 6y = 0$$
- *All of three equations are legendre's equation*
- *None of other*

Question No. 11 **Marks : 2**

The differential equation $(2x^2y - 2x^3)dy + (4x^3 - 6x^2y + 2xy^2)dx = 0$ is

- Separable
- Exact
- Linear
- Bernoulli's

FINALTERM EXAMINATION

SPRING 2007

MTH401 - DIFFERENTIAL EQUATIONS (Session - 4)

Marks: 55

Time: 150min

StudentID/LoginID: ______________________________

Student Name: ______________________________

Center Name/Code: ______________________________

Exam Date: Tuesday, July 10, 2007

Please read the following instructions carefully before attempting any of the questions:

1. Symbols by using math type should be pasted on the paper direct from the math type not from the word document otherwise it would not be visible.

2. Do not ask any questions about the contents of this examination from anyone.

   a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

   b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

   c. Write all steps, missing steps may lead to deduction of marks.

3. This examination is closed book and closed notes.

4. Use of Calculator is allowed.

5. Attempt all questions. Marks are written adjacent to each question.

**WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will be awarded grade `F` in this course.

For Teacher's use only

Question	1	2	3	4	5	6	7	8	9	10	Total

Marks											
Question	11										
Marks											

Question No: 1 (Marks: 2) - Please choose one

The Wronskian of the function $W(e^{x}, e^{-x}) =$ ______ is

▶ 1

▶ 0

▶ -2

▶ None of these

Question No: 2 (Marks: 2) - Please choose one

The eigen values of matrix $\begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$ are

▶ $\lambda = 0, -1$

▶ $\lambda = 4, -1$

▶ $\lambda = 1, 5$

▶ None of these

Question No: 3 (Marks: 2) - Please choose one

Roots of the equation $y''' + y' = 0$ will be

▶ 0,1,2

▶ 0,+ i , - i

▶ 0,1,i

▶ None of these

Question No: 4 (Marks: 2) - Please choose one

A differential equation $y'' + 16y = 0 \ \ with \ \ y(0) = 0, \ y(\frac{\pi}{8}) = 0$ is called

▶ Initial value problem

▶ Boundary value problem

▶ None of these

Question No: 5 (Marks: 2) - Please choose one

Suppose the functions $f_1(x), f_2(x), ..., f_n(x)$ possess at least n-1 derivatives on interval I , if $W(f_1, f_2, ..., f_n) \neq 0$ is called _________

▶ Linearly dependent

▶ Linearly independent

▶ None of these

Question No: 6 (Marks: 5)

Solve the initial value problem

$$2(y-1)\,dy = (3x_2 + 4x + 2)\,dx \quad , \quad y(0) = -1$$

Question No: 7 (Marks: 10)

Solve

$$\frac{dy}{dx} + \frac{xy}{1-x_2} = x\,y^{\frac{1}{2}}$$

Question No: 8 (Marks: 5)

Find the general solution of the given differential equation on $(0, \infty)$

$$25\,x_2\,y_{//} + 25\,xy_{/} + (25\,x_2 - 1)\,y = 0$$

Question No: 9 (Marks: 10)

Use the Gauss-Jordan elimination method to solve the linear system

$5x - 2y + 4z = 10$

$x + y + z = 9$

$4x - 3y + 3z = 1$

Question No: 10 (Marks: 10)

Derive the expression of $J_n(x)\ \ for\ \ n = \frac{1}{2}$

Question No: 11 (Marks: 5)
Solve by using Cauchy Euler method $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + 4y = 0$

[redacted]

[redacted]

[redacted]

Question No: 4 **Marks:2**

The given functions $f_1(x) = 1 + x, f_2(x) = x, f_2(x) = x^2$ are linearly independent.

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Question No: 5 **Marks: 2**

A set of functions whose wronskian is zero guarantees that set of functions is linearly dependent.

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Question No: 6 **Marks: 10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$(r\theta - 4r + \theta - 4)dr - \left(r^2\theta + 20r^2 - \theta - 20\right) d\theta = 0$$

Solution:

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form d y / d x = h(x)g(y)

$$(r\theta - 4r + \theta - 4)dr - \left(r^2\theta + 20r^2 - \theta - 20\right) d\theta = 0$$

$$(r\theta - 4r + \theta - 4)dr = \left(r^2\theta + 20r^2 - \theta - 20\right) d\theta$$

$$\left[r(\theta - 4) + 1(\theta - 4)\right] \quad r^2(\theta + 20) - 1(\theta + 20)\ d\theta$$

$$dr =$$

$$(r + 1)(\theta - 4)\, dr = \left(r^2 - 1\right)(\theta + 20)\, d\theta$$

$$\frac{(r + 1)}{(r^2 - 1)} dr = \frac{(\theta + 20)}{(\theta - 4)} d\theta$$

$$\frac{1}{r - 1} dr = \frac{(\theta + 20)}{(\theta - 4)} d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$\left(2y^2 + 3x\right) dx + 2xydy = 0$$

$(2y^2 + 3x)\ dx + 2xydy = 0$

$M(x, y) = 2y^2 +$ $N(x, y) = 2xy$

$3x,$

$M_y = 4$ $N_x = 2y$

$y,$

$M_y \neq N_x$

Thus it is not exact now we apply techniques to make it exact

$(2y^2 + 3x)\ dx + 2xydy = 0$

$$\frac{M_y - N_x}{N} = \frac{4y - 2y}{2xy} = \frac{2y}{2xy} = \frac{1}{x}$$

$$I.F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$(2xy^2 + 3x^2)\ dx + 2x^2ydy = 0$

$M(x, y) = 2xy_2 +$ $N(x, y) = 2x_2y$

$3x_2,$

$M_y = 4xy, N_x =$

$4xy$

$M_y = N_x$

Which shows that now equation is exact

(a) Solve given Bernoulli equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ **(Just make the given equation linear in v, do not integrate)**

$$xy - \frac{dy}{dx} = y^3 e^{-x2}$$

$$xy^{-2} - \frac{dy}{dx} y^{-3} = e^{-x_2}$$

$put\ y_{-2} = v$

$$-2\frac{dy}{dx} y^{-3} = \frac{dv}{dx}$$

$$-\frac{dy}{dx} y^{-3} = \frac{1}{2}\frac{dv}{dx}$$

Then

$$\frac{1}{2}\frac{dv}{dx} + vx = e^{-x^2}$$

$$\frac{dv}{dx} + 2vx = 2e^{-x^2}$$

Thus it is linear in “v”.

[redacted]

[redacted]

(b) The population of a town grows at rate proportional to the population at any time. Its initial population of 100 decreased by 20% in 20 years what will be the population in 30 years? **(Just make the model of the population dynamics as well as just describe the given conditions do not solve further)**

Solution……………………………………………………………………………………………………

Suppose that P_0 is the initial population of the town, as given P_0 is 100 and P(t) the population at any time t then population growth by the differential equation

$$\frac{dP}{dt} \alpha P$$

$$\frac{dP}{dt} = kP$$

$$\frac{dP}{P} = kdt$$

Integrate both sides

$$\ln P = kt + c$$

$$P = e^{kt + c}$$

$$P = e^{kt} e^{c}$$

$$P = P_0 e^{kt} \qquad say\ P_0 = e^{c}$$

Where P_0 is the initial population of the town

P_0 =100 = P(0)

P(20)=100-$\frac{20}{100}(100)$

P(20)=80

P (30) =?

Question No: 8 **Marks: 10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)**.

$$x^2y'' - 5xy' + 9y = \quad y_1 = x^3 \ln x$$

[redacted]

$$x^2 y'' - 5xy' + 9y = 0;\ y_1 = x^3 \ln x$$

differential equation can be written as

$$x^2 y'' - \frac{5}{x} xy' + \frac{9}{x^2} = 0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int p dx}}{y_1^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{-\int -\frac{5}{x} dx}}{\left(x^3 \ln x \right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\int \frac{5}{x} dx}}{\left(x^3 \ln x \right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{5 \ln x}}{\left(x^3 \ln x \right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\ln x^5}}{\left(x^3 \ln x \right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{x^5}{x^6 \left(\ln x \right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{(\ln x)^{-2}}{x} dx$$

$$y_2 = x^3 \ln x \frac{(\ln x)^{-2+1}}{-2+1}$$

[redacted]

x

$$y_2 = -x^3$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = 2\cos\theta$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$$y_p = A\cos\theta + B\sin\theta$$

Then

$$y'_p = -A\sin\theta + B\cos\theta$$

$$y''_p = -A\cos\theta - B\sin\theta$$

Substituting in the given differential equation

$$-A\cos\theta - B\sin\theta - 2\left(-A\sin\theta + B\cos\theta\right) - 3\left(A\cos\theta + B\sin\theta\right) = 2\cos\theta$$

$$\left(-A - 2B - 3A\right)\cos\theta + \left(-B + 2A - 3B\right)\sin\theta = 2\cos\theta$$

From the resulting equations

$-A-2B-3A=2;$ $\quad -B+2A-3B=0$

$-4A-2B=2;$ $\quad 2A-4B=0$

$2A+B=-1;$ $\quad A-2B=0 \rightarrow A=2B$

$\rightarrow 2(2B)+B=-1$

$\rightarrow 5B=-1$

$\rightarrow B=\frac{-1}{5}$

$\rightarrow A=2\left(\frac{-1}{5}\right)=\frac{-2}{5}$

[redacted]

(a) Solve the differential equation by the undetermined coefficient **(annihilator operator).**

$y''-4y'=x_3-2x+1$

If complimentary solution is given below

$y_c=c_1+c_2e^{4x}$

Then find general solution.

Solution……………………………………………………………………………………………

In this case of input function is

g(x)= x^3-2x+1

further

$D_4(g(x))=D_3(x_3-2x+1)=0$

Therefore the differential operator D^4 annihilates the function g. operating on both sides

$D_4(D_2-4D)y=D_4(x_3-2x+1)$

$D^4(D^2-4D)y=0$

This is the homogeneous equation of order 5. Next we solve this higher order equation.

Thus auxiliary equation is

$m_4(m_2-4m)=0$

$m_5(m-4)=0$

$m=0,0,0,0,0,4$

Thus its general solution of the differential equation must be

$y=c_1+c_2x+c_3x^2+c_4x^3+c_5x^4+c_6e^{4x}$

(b) Solve the differential equation by the variation of parameters

$y''-2y'-3y=x_3$

If complimentary solution is given below

$y_c = c_1 e^{3x} + c_2 e^{-x}$

Then just find particular solution **(do not integrate)**.

Solution………

$y'' - 2y' - 3y = x^3$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^3

We construct the determinants

Since $y_1 = e^{3x}$, $y_2 = e^{-x}$ so

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & e^{-x} \\ 3e^{3x} & -e^{-x} \end{vmatrix} = -e^{3x-x} - 3e^{3x-x} = -4e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ x^3 & -e^{-x} \end{vmatrix} = -x^3 e^{-x}$$

$$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & x^3 \end{vmatrix} = x^3 e^{3x}$$

We determine the derivatives of the function u_1 and u_2

$$u'_1 = \frac{W_1}{W} = \frac{-x^3 e^{-x}}{-4e^{2x}} \to u_1 = \int \frac{x^3}{4} e^{-3x} dx$$

$$u'_2 = \frac{W_2}{W} = \frac{x^3 e^{3x}}{-4e^{2x}} \to u_2 = \int \frac{x^3}{4} e^{x} dx$$

$$y_p = u_1 e^{3x} + u_2 e^{-x}$$

is required particular solution

[redacted]

[redacted]

[redacted]

Population dynamics are not practical application of the first order differential equations.

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Question No: 4	Marks: 2

A set

$$\{ y_1, y_2, \ldots, y_n \}$$

Of n linearly dependent solutions, on interval I, of the homogeneous linear nth-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Is said to be a fundamental set of solutions on the interval I.

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Question No: 5	Marks: 2

The differential operator that annihilates $10x^3 - 2x$ is D^4.

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Question No: 6	Marks:10

(a) Define separable form. Just separate the variables of the given differential equation.

$$(3r\theta - 3\theta + r - 1)\, dr - (2r\theta + 4\theta - r - 2)\, d\theta = 0$$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$$(3r\theta - 3\theta + r - 1)\, dr - (2r\theta + 4\theta - r - 2)\, d\theta = 0$$

$$(3r\theta - 3\theta + r - 1)\, dr = (2r\theta + 4\theta - r - 2)\, d\theta$$

$$[3\theta(r - 1) + 1(r - 1)]\, dr = [2\theta(r + 2) - 1(r + 2)]\, d\theta$$

$$(r - 1)(3\theta + 1)\, dr = (r + 2)(2\theta - 1)\, d\theta$$

$$\frac{(r - 1)}{(r + 2)}dr = \frac{(2\theta - 1)}{(3\theta + 1)}d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further)**.

$$\left(\frac{3y^2 - x^2}{y^5}\right)\frac{dy}{dx} + \frac{x}{2y^5} = 0$$

Solution

It can also be written as

$$xdx + 2\left(3y^2 - x^2\right)dy = 0$$

$$M(x, y) = x,\quad N(x, y) = 6y^2 - 2x^2$$

$$M_y = 0,\quad N_x = -4x$$

$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$xdx + \left(6y^2 - 2x^2\right)dy = 0$$

$$\frac{N_x - M_y}{M} = \frac{-4x}{x} = -4 = g(y)$$

$$I.F = e^{\int -4\,dy} = e^{-4y}$$

$$xe^{-4y}dx + e^{-4y}\left(3y^2 - 2x^2\right)dy = 0$$

$$M(x, y) = xe^{-4y},\quad N(x, y) = e^{-4y}\left(3y^2 - 2x^2\right)$$

$$M_y = -4xe^{-4y},\ N_x = -4xe^{-4y}$$

$$M_y = N_x$$

Which shows that equation is exact

Question No: 7 **Marks:10**

(a) Solve the Bernoulli equation $x^3\frac{dy}{dx} + 2xy = y^5$ **(Just make the given equation linear in v, do not integrate)**

Solution

$$x^3\frac{dy}{dx} + 2xy = y^5$$

$$\frac{dy}{dx}y^{-5} + \frac{2}{x^2}y^{-4} = \frac{1}{x^3}$$

$$put\ y^{-4} = v$$

$$-4\frac{dy}{dx}y^{-5} = \frac{dv}{dx}$$

$$\frac{dy}{dx}y^{-5} = -\frac{1}{4}\frac{dv}{dx}$$

Then

$$-\frac{1}{4}\frac{dv}{dx} + \frac{2v}{x^2} = \frac{1}{x^3}$$

$$\frac{dv}{dx} - \frac{8v}{x^2} = \frac{-4}{x^3}$$

Thus it is linear in “v”.

(b) Initially there were 200 milligrams of a radioactive substance present. After 8 hours the mass increased by 4%. If the rate of decay is proportional to the amount of the substance present at any time, determine half-life of the radioactive substance? **(Just make the model of the radioactive decay as well as describe the given conditions do not solve further)**

Solution

Suppose that A_0 is the initial amount, as given A_0 is 200 and A(t) be the amount present at time t then its governed by the differential equation

$$\frac{dA}{dt} \alpha A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = kdt$$

Integrate both sides

$$\ln A = kt + c$$

$$A = e^{kt+c}$$

$$A = e^{kt}e^{c}$$

$$A = A_0 e^{kt} \qquad say\ A_0 = e^{c}$$

Where A_0 is the initial amount

$A_0 = 200 = A(0)$

$A(8) = 200 + \frac{4}{100}(200)$

$A(8) = 208$

$A(T) = 100$

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae).** $(1+2x)y'' + 2xy' - 4 = 0$, $y_1 = e^{-2x}$

$$(1+2x)y'' + 2xy' - 4 = 0 \quad , \quad y_1 = e^{-2x}$$

differential equation can be written as

$$y'' + \frac{2x}{1+2x}y' - \frac{4}{1+2x} = 0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int p dx}}{y_1^2} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{2x}{1+2x} dx}}{\left(e^{-2x}\right)^2} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{1+2x-1}{1+2x} dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\int \left(\frac{1}{1+2x} - 1\right) dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-x} e^{\ln(1+2x)}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{2\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int (1+2x) e^{3x} dx$$

$$y_2 = e^{-2x} \left[(1+2x) \frac{e^{3x}}{3} - \int (1+2x)' \frac{e^{3x}}{3} dx \right]$$

$$y_2 = e^{-2x} \left[(1+2x) \frac{e^{3x}}{3} - \int \frac{2e^{3x}}{3} dx \right]$$

$$y_2 = (1+2x)\frac{e^x}{3} - \frac{2e^x}{9}$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = e^{4x}$$

Solution

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$y_p = Ae^{4x}$

Then

$y'_p = 4Ae^{4x}$

$y''_p = 16Ae^{4x}$

Substituting in the given differential equation

$$16Ae_{4x} - 2\left(4Ae_{4x}\right) - 3Ae_{4x} = e_{4x}$$

$$e_{4x}\left(16A - 8A - 3A\right) = e_{4x}$$

From the resulting equations

$$5A = 1$$

$$A = \frac{1}{5}$$

$$y^p = \frac{1}{5}e^{4x}$$

Question No: 9 Marks: 10

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x\cos x$$

If complimentary solution is given below

$$y_c = c_1 + c_2e^{4x}$$

Then just find general solution.

Solution

In this case of input function is

g(x)= $x\cos x$

further

$$(D_2 + 1)_2(g(x)) = (D_2 + 1)_2(x\cos x) = 0$$

Therefore the differential operator $(D_2 + 1)_2$ annihilates the function g. operating on both sides

$$(D_2 + 1)_2(D_2 - 4D)y = (D_2 + 1)_2(x\cos x)$$

$$(D_2 + 1)_2(D_2 - 4D)y = 0$$

This is the homogeneous equation of order 6. Next we solve this higher order equation.

Thus auxiliary equation is

$(m^2 + 1)^2 (m^2 - 4m)\, y = 0$

$m(m - 4)(m^2 + 1)^2 = 0$

$m = 0, 4, i, i, -i, -i$

Thus its general solution of the differential equation must be

$y = c_1 + c_2 e^{4x} + (c_3 + c_4 x)\cos x + (c_5 + c_6 x)\sin x$

(b) Solve the differential equation by the variation of parameters

$y'' - y = x^2$

Complimentary solution is given below If

$y_c = c_1 e^{x} + c_2 e^{-x}$

Then just find particular solution **(do not integrate).**

Solution

$y'' - y = x^2$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^2

We construct the determinants

Since $y_1 = e^{x}$, $y_2 = e^{-x}$ so

$$W(y_1, y_2) = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -e^{x-x} - e^{x-x} = -2$$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ x^2 & -e^{-x} \end{vmatrix} = -x^2 e^{-x}$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & x^2 \end{vmatrix} = x^2 e^x$$

We determine the derivatives of the function u_1 and u_2

$$u_1' = \frac{W_1}{W} = \frac{-x^2 e^{-x}}{2} \rightarrow u_1 = \int \frac{x^2}{2} e^{-x}\,dx$$

$$u_2' = \frac{W_2}{W} = \frac{x^2 e^{-x}}{2} \rightarrow u_2 = -\int \frac{x^2}{2} e^{-x}\,dx$$

$y_p = u_1 e^{x} + u_2 e^{-x}$

is required particular solution

[redacted]

[redacted]

[redacted]

[redacted]

The given functions $f_1(x) = 5, f_2(x) = \cos^2 x, f_3(x) = \sin^2 x$ are linearly independent.

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Question No: 5 **Marks: 2**

A set of functions whose wronskian is zero then set of functions may or may not be dependent.

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Question No: 6 **Marks: 10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$\sec y \frac{dy}{dx} + \sin(x - y) = \sin(x + y)$$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$$\sec y \frac{dy}{dx} + \sin(x - y) = \sin(x + y)$$

$$\sec y \frac{dy}{dx} = \sin(x + y) - \sin(x - y)$$

$$\sec y \frac{dy}{dx} = \sin(x + y) - \sin(x - y)$$

$$\sec y \frac{dy}{dx} = \sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y$$

$$\sec y \frac{dy}{dx} = 2 \cos x \sin y$$

$$\frac{dy}{2 \cos y \sin y} = \cos x dx$$

$$\frac{dy}{\sin 2y} = \cos x dx$$

$$\operatorname{cosec} 2y\, dy = \cos x dx$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$e^x dx + \left(e^x \cot y + 2y \operatorname{cosec} y\right) dy = 0$$

Solution

$e_x dx + \left(e_x \cot y + 2y\operatorname{cosec} y\right) dy = 0$

$M(x, y) = e_x,\ N(x, y) = e_x \cot y + 2y\operatorname{cosec} y$

$M_y = 0,\ N_x = e^x \cot y$

$M_y \neq N_x$

Thus it is not exact now we apply techniques to make it exact

$e_x dx + \left(e_x \cot y + 2y\operatorname{cosec} y\right) dy = 0$

$\frac{N_x - M_y}{M} = \frac{e^x \cot y - 0}{e^x} = \cot y$

$I.F = e^{\int \cot y dy} = e^{\ln \sin y} = \sin y$

$e_x \sin y dx + \left(e_x \cos y + 2y\right) dy = 0$

$M(x, y) = e_x \sin y,\ N(x, y) = e_x \cos y + 2y$

Question No: 7 **Marks: 10**

(a) Solve Bernoulli equation $y + 2\frac{dy}{dx} = y^3(x - 1)$ **(Just make the given equation linear in v, do not integrate)**

Solution

$xy - \frac{dy}{dx} = y^3(x - 1)$

$xy^{-2} - \frac{dy}{dx} y^{-3} = (x - 1)$

$put\ y_{-2} = v$

$-2\frac{dy}{dx} y^{-3} = \frac{dv}{dx}$

$-\frac{dy}{dx} y^{-3} = \frac{1}{2}\frac{dv}{dx}$

Then

$\frac{1}{2}\frac{dv}{dx} + vx = (x - 1)$

$\frac{dv}{dx} + 2vx = 2(x - 1)$

(b) The radioactive isotope of the lead, Pb-209, decay at a rate proportional to the amount present at any time and has a half-life of 4 hours. If 2 grams of the lead is present initially, how long will it take for 80% of the lead to decay? **(Just make the model of the radioactive decay as well as describe the given conditions do not solve further)**

Solution

Suppose that A_0 is the initial amount of isotope, as given A_0 is 100 and A (t) be the amount present at time t it governed by the differential equation.

$$\frac{dA}{dt} \alpha A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = kdt$$

Integrate both sides

$$\ln A = kt + c$$

$$A = e_{kt + c}$$

$$A = e^{kt} e^{c}$$

$$A = P_0 e^{kt} \qquad say\ A_0 = e^{c}$$

Where A_0 is the initial amount of isotope

A_0 =2 = A(0)

A(4)=2/2=1

Then we have to find time when radioactive isotope will take 80% decay. So as A initially given 2 and 80% of 2 is 8/5 so decay would be 1-8/5 = -3/5

P (t) = -3/5 , t=?

Question No: 8 **Marks:10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)**. $x^2 y'' - 4xy' + 6y = \quad y_1 = x^2$

Solution

0;

$x_2 y'' - 4xy' + 6y = 0_1; y = x_2$

differential equation can be written as

$$y'' - \frac{4}{x} xy' + \frac{6}{x^2} = 0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int pdx}}{y_1^2} dx$$

$$y_2 = x^2 \int \frac{e^{-\int -4/x \, dx}}{\left(x^2\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\int 4/x \, dx}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^2 \int \frac{e^{4 \ln x}}{x^4} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\ln x^4}}{x_4} dx$$

$$y_2 = x^2 \int \frac{x^4}{x_4} dx$$

$$y_2 = x^2 \int dx$$

$$y_2 = x^3$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = 4 \sin \theta$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Solution

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$$y_p = A \cos\theta + B \sin\theta$$

Then

$$y'_p = -A \sin\theta + B \cos\theta$$

$$y''_p = -A \cos\theta - B \sin\theta$$

Substituting in the given differential equation

$$-A \cos\theta - B \sin\theta - 2\left(-A \sin\theta + B \cos\theta\right) - 3\left(A \cos\theta + B \sin\theta\right) = 4 \sin\theta$$

$$\left(-A - 2B - 3A\right) \cos\theta + \left(-B + 2A - 3B\right) \sin\theta = 4 \sin\theta$$

From the resulting equations

$-A - 2B - 3A = 0;$ $\qquad -B + 2A - 3B = 4$

$-4A - 2B = 0;$ $\qquad 2A - 4B = 4$

$2A + B = 0;$ $\qquad A - 2B = 2 \rightarrow A = 2 + 2B$

$\rightarrow 2(2 + 2B) + B = 0$

$\rightarrow 4 + 5B = 0$

$\rightarrow B = \frac{-4}{5}$

$\rightarrow A = 2 + 2\left(\frac{-4}{5}\right) = \frac{2}{5}$

$y_p = \frac{2}{5}\cos\theta - \frac{4}{5}\sin\theta$

[redacted]

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = e^{2x}$

If complimentary solution is given below

[redacted]

e

Then just find general solution.

In this case of input function is

g(x)= e^{2x}

further

$(D-2)(g(x)) = (D-2)(e_{2x}) = 0$ annihilates the function g. operating on both sides

Therefore the differential operator D^3

$(D-2)(D_2 - 4D)y = (D-2)(e_{2x})$

$(D-2)(D^2 - 4D)y = 0$

This is the homogeneous equation of order 3. Next we solve this higher order equation.

Thus auxiliary equation is

$(m-2)(m_2 - 4m) = 0$

$m(m-2)(m-4) = 0$

$m = 0, 2, 4$

Thus its general solution of the differential equation must be

$y = c_1 + c_2 e^{4x} + c_3 e^{2x}$

(b) Solve the differential equation by the variation of parameters

$y'' - 4y + 3 = \cos x$

If complimentary solution is given below

$y_c = c_1 e^{3x} + c_2 e^{x}$

Then just find particular solution **(do not integrate).**

Solution

$y'' - 4y + 3 = \cos x$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = cos x

We construct the determinants

Since $y_1 = e^{3x}$, $y_2 = e^{x}$ so

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & e^{x} \\ 3e^{3x} & e^{x} \end{vmatrix} = e^{3x+x} - 3e^{3x+x} = -2e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & e^{x} \\ \cos x & e^{x} \end{vmatrix} = \cos xe^{x}$$

$$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \cos x \end{vmatrix} = \cos xe^{3x}$$

We determine the derivatives of the function u_1 and u_2

$$u'_1 = \frac{W_1}{W} = \frac{\cos xe^{x}}{-2e_{4x}} \to u_1 = \int \frac{\cos x}{-2} e^{-3x} dx$$

$$u'_2 = \frac{W_2}{W} = \frac{\cos xe^{3x}}{-2e_{4x}} \to u_2 = \int \frac{\cos x}{-2} e^{-x} dx$$

$y_p = u_1 e^{3x} + u_2 e^{x}$

is required particular solution

[redacted]

[redacted]

[redacted]

[redacted]

$f_1(x) = 1 + x, f_2(x) = x, f_3(x) = x^2$

Question No: 5 **Marks:2**

The differential operator that annihilates $10x^3 - 2x$ is:

D^4

Question No: 6 **Marks:10**

Solve the following differential equation by using an appropriate substitution.

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

Solution

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 + x^2}{xy}$$

$$\Rightarrow \quad \text{Homogeneous equation, so put } y = vx, \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2x^2 + x^2}{x^2v}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \frac{1}{v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1}{v} \quad \Rightarrow \quad vdv = \frac{1}{x}dx$$

$$\Rightarrow \quad \int vdv = \int \frac{1}{x}dx \quad \Rightarrow \quad \frac{v^2}{2} = \ln x + \ln C$$

$$\Rightarrow \quad \frac{y^2}{x^2} = 2\ln xC$$

Question No: 7 **Marks:10**

The population of a town grows at a rate proportional to the population at any time. Its initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?

Solution:

Let $P(t)$ be the population at any time t, then rate of grows will be

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \quad \frac{dP}{dt} = kP$$

Here k is constant of proportionality. Since initially population was 500, therefore $P(0) = 500$. Also this population increases by 15% in 10 years. The 15% of 500 is $\frac{15}{100}(500) = 75$, therefore population after 10 years is (initial population + increase in 10 years) = 500+75 = 575 i.e. $P(10) = 575$. So we have the boundary value problem

$\frac{dP}{dt} = kP$ subject to boundary conditions $P(0) = 500, P(10) = 575$.

This first order differential equation. Its solution is given by

$P = Ce^{kt}$ where C is constant of integration.

Applying boundary conditions, we get $C = 500,\ k = 0.0139$. So the solution is

$$P(t) = 500e^{(0.0139)t}$$

Thus population after 30 years is obtained by putting $t = 30$ in above equation i.e.

$$P(30) = 500e_{(0.0139)30}$$
$$\approx 760.$$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$x^2y'' + 2xy' - 6y = 0;$ $\quad y_1 = x^2$

Solution: Comparing with $y'' + P(x)y' + Q(x)y = 0$

Here $P(x) = \frac{2}{x}$. But second solution is given be

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$\Rightarrow \quad = x^2 \int \frac{e^{-\int \frac{2}{x}\,dx}}{x^4}\,dx$$

$$\Rightarrow \quad = x_2 \int \frac{e^{-2\ln x}}{x^4}\,dx = x^2 \int \frac{e^{\ln x^{-2}}}{x^4}\,dx$$

$$\Rightarrow \quad = x^2 \int \frac{x^{-2}}{x^4}\,dx = x_2 \int \frac{1}{x^6}\,dx$$

$$\Rightarrow \quad = x^2 \left(\frac{x^{-5}}{-5} \right)$$

$$\Rightarrow \quad = -\frac{1}{5x^3}$$

Note: This question can also solve using other method.

[redacted]

[redacted]

[redacted]

[redacted]

Question No: 5	Marks:2

The differential operator that annihilates $4e^{x/2}$ is:

$2D-1$

Question No: 6	Marks:10

Solve the following differential equations.

$$\left(1+\ln x+\frac{y}{x}\right)dx=\left(1-\ln x\right)dy$$

Solution:

Here

$$M=1+\ln x+\frac{y}{x}\,,\quad N=-\left(1-\ln x\right)$$

$$\Rightarrow\qquad M_y=\frac{\delta M}{\delta y}=\frac{1}{x}\,,\quad N_x=\frac{\delta N}{\delta x}=\frac{1}{x}$$

$$\Rightarrow\qquad M_y=N_x$$

So the given equation is an exact equation. Thus there exists a function $f\left(x,y\right)$ such that

$$\frac{\delta f}{\delta x}=M \qquad and \qquad \frac{\delta f}{\delta y}=N$$

$$\Rightarrow\ \frac{\delta f}{\delta x}=1+\ln x+\frac{y}{x}\ --(1)\quad and \qquad \frac{\delta f}{\delta y}=\ln x-1--(2)$$

$$(1)\ \Rightarrow\ f=x+x\ln x-x+y\ln x+H(y)=x\ln x+y\ln x+H(y)$$

$$\Rightarrow\ \frac{\delta f}{\delta y}=\ln x+H'(y)$$

$$(2)\Rightarrow \ln x-1=\ln x+H'(y)$$

$$\Rightarrow\qquad -1=H'(y)$$

$$\Rightarrow\qquad H(y)=-y$$

$$Hence\quad f\left(x,y\right)=x\ln x+y\ln x-y$$

Question No: 7	Marks:10

Initially there were 100 milligrams of a radioactive substance present. After 6 hours the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours.

Solution:

Let $A(t)$ be amount present at any time t. Then by given conditions, we have

$$\frac{dA}{dt} \propto A$$

$$\Rightarrow \quad \frac{dA}{dt} = kA$$

Initially there were 100 milligrams, therefore $A(0) = 100$. Moreover, decreased by 3% will give us $100 - \frac{3}{100}(100) = 97$ milligrams after 6 hours i.e. $A(6) = 97$. So we have boundary value problem

$$\frac{dA}{dt} = kA \text{ subject to boundary conditions } A(0) = 100,\ A(6) = 97$$

The solution of this equation is given by

$A(t) = Ce^{kt}$ where C is constant of integration.

Applying boundary conditions, we get

$$C = 100, \quad k = -0.005076$$

$$\Rightarrow \quad A(t) = 100e^{-0.005076t}$$

Amount remaining after 24 hours is obtained by putting $t = 24$ in above equation i.e.

$$\Rightarrow \quad A(t) = 100e^{-0.005076(24)}$$

$$= 88.529 \quad mg.$$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$$x^2 y'' + y' = 0; \quad y_1 = \ln x$$

Solution:

Comparing this equation with $y'' + P(x)y' + Q(x)y = 0$, we get

$$P(x) = \frac{1}{x^2} .$$

But second solution is given by

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2} dx$$

$$\Rightarrow \qquad y_2 = \ln x \int \frac{e^{-\int \frac{1}{x^2}dx}}{(\ln x)^2}\,dx$$

$$= \ln x \int \frac{e^{\frac{1}{x}}}{(\ln x)^2}\,dx$$

This is the answer.

Note: This question can also solve using second method.

[redacted]

[redacted]

[redacted]

[redacted]

Question No: 5 **Marks:2**

The differential operator that annihilates $4e^{2x}$ is:

$(D-2)(D+5)$

Question No: 6 **Marks:10**

Find the general solution of the given differential equation.

$$\frac{dy}{dx}+2xy=x^3$$

Solution:

It is of the form $\frac{dy}{dx}+P(x)y=Q(x)$ i.e. Linear First Order Differential Equation with

$P(x)=2x,\ Q(x)=x^3$

Thus integration factor is given by

$$I.F=u(x)=e^{\int P(x)dx}$$

$$\Rightarrow \quad =e^{\int 2xdx}=e^{x^2}$$

But the solution in this case is

$$y=\frac{\int u(x)Q(x)dx+C}{u(x)}\text{ --------------------(1.}$$

Now

$$\int u(x)Q(x)dx=\int x^3e^{x^2}$$

$$=\frac{1}{2}\int\left(e^{x^2}2x\right)x^2dx$$

$$=\frac{1}{2}\left\{e^{x^2}x^2-\int e^{x^2}2xdx\right\}\quad \text{integration by parts}$$

$$y = \frac{\frac{1}{2}\left\{x^2 - 1\right\} e^{x^2} + C}{e^{x^2}}$$

Question No: 7 **Marks:10**

A thermometer is taken from an inside room to the outside where the air temperature is 5° F. After 1 minute the thermometer reads 55° F, and after 5 minutes the reading is 30° F. What is the initial temperature of the room?

Solution:

Let $T(t)$ be temperature at any time t and T_0 be the temperature of the surroundings. Then by Newton's Method, we know that

$$\frac{dT}{dt} = k\left(T - T_0\right)$$

Where k is constant of proportionality. Here we are given $T_0 = 5$ and $T(1) = 55, T(5) = 30$. Solving above equation we get

$$T = T_0 + Ce^{kt}$$

$$\Rightarrow \quad T = 5 + Ce^{kt}$$

Using above conditions we get

$$k = -0.173\ ,\ C \approx 59.44\ .$$

So the initial temperature is given by

$$T = 5 + Ce^{0}$$
$$= 5 + C$$
$$\approx 5 + 59.44 = 64.44^{\circ}\ F\ .$$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$$4x^2 y'' + y = 0;\ y_1 = x^{1/2} \ln x$$

Solution:

Comparing this equation with $y'' + P(x)\,y' + Q(x)\,y = 0$, we get

$$P(x) = 0 .$$

As second solution is given by

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

Using given conditions, we get

$$y_2 = x^{1/2}\ln x \int \frac{e^0}{\left(x^{1/2}\ln x\right)^2}\,dx$$

$$= x^{1/2}\ln x \int \frac{1}{x(\ln x)^2}\,dx$$

$$= x^{1/2}\ln x \int \frac{1}{(\ln x)^2}\frac{1}{x}\,dx$$

$$= x^{1/2}\ln x \left(\frac{(\ln x)^{-1}}{-1}\right)$$

$$= -x^{1/2}$$

MTH401 Deferential Equations

Mid Term Examination – Spring 2006

Time Allowed: 90 Minutes

1. The duration of this examination is 90 minutes.
2. Symbols by using math type should be pasted on the paper direct from the math type not from the word document otherwise it would not be visible.
3. Do not ask any questions about the contents of this examination from anyone.
   - *a*. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.
   - *b*. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.
   - *c*. Write all steps, missing steps may lead to deduction of marks.
4. This examination is closed book and closed notes.
5. Use of Calculator is allowed.
6. Attempt all questions. Marks are written adjacent to each question.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will be awarded grade `F` in this course.**

Question No. 1	**Marks : 1**

The method of undetermined coefficient is limited to homogeneous linear differential equation

True
False

Question No. 2 **Marks : 1**

In the homogeneous differential equation after substitution v=y/x the equation reduces to.

- Separable differential equation.
- Exact differential equation.
- Remain homogeneous equation.
- None of the other

Question No. 3 **Marks : 10**

Solve the differential equation by the variation of parameters

$$y'' - 9y' + 9y = xe^{3x}$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 x e^{3x}$$

Then just find the particular solution.

Question No. 4 **Marks : 5**

Determine whether the functions are linearly independent or dependent on $(-\infty, \infty)$

$f_1(x) = \quad f_2(x) = x, f_3(x) = e$
0,

Question No. 5 **Marks : 10**

Solve

$$\frac{dy}{dx} + xy = xy^2$$

Question No. 6 **Marks : 10**

Solve the differential equation by integrating factor technique

$$y^2 dx + xydy = 0$$

Question No. 7 **Marks : 1**

If the Wronskian W of three function f(x),g(x),h(x) is zero, what can be said about the dependency of the functions

- May or may not be dependent
- Always dependent
- Never dependent
- None of the other

Question No. 8	**Marks : 1**

If $a_n(x) = 0$ in the differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2} y}{dx^{n-2}} + \ldots. + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

for some $x \in I$ then

I. Solution of initial value problem may not unique.
II. Solution of initial value problem may not even exist.
III. Solution of initial value problem should exist.
IV. Solution of initial value problem is unique.

- I is correct only
- I and II are correct
- I and III are correct
- IV is correct only

Question No. 9	**Marks : 1**

Equation of the form $\frac{dy}{dx} + y = x^2 y^2$ is called

- First order linear differential equation
- Bernoulli equation
- Separable equation
- None of the other.

	FINALTERM EXAMINATION FALL 2006 MTH401 - DIFFERENTIAL EQUATIONS (Session - 1)	 Marks: 53 Time: 120min

StudentID/LoginID: ______________________________

Student Name: ______________________________

Center Name/Code: ______________________________

Exam Date: Monday, February 12, 2007

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.

2. Do not ask any question about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the question, attempt it to the best of your understanding.

b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

c. Write all steps, missing steps may lead to deduction of marks.

3. Calculator is allowed.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

For Teacher's use only

Question	1	2	3	4	5	6	7	8	9	10	Total
Marks											
Question	11										
Marks											

Question No: 1 (Marks: 2) - Please choose one

If the variation of the path of the curves can be describrd by the concept of differential equations then which of the following differential equation describe the path for $y-axis$.

- $\frac{dy}{dx} = 1$
- $\frac{dy}{dx} = 0$
- $\frac{dy}{dx} = -1$
- $\frac{dy}{dx} = \infty$

Question No: 2 (Marks: 2) - Please choose one

Suggestive form of the constant input function for the non homogeneous differential equation under the method entitled as "**Method of the undetermined coefficient**" is

- $f(x) = e_x$
- $f(x) = a$
- $f(x) = e_{ax}\left(ACosx + BSinx\right)$
- Suggestive form is impossible.

Question No: 3 (Marks: 2) - Please choose one

Which of the following function is linearly dependant to the exponential function e^x ?

- ► $-e_x$
- ► e_{-x}
- ► xe^x
- ► $-xe_{-x}$

Question No: 4 (Marks: 2) - Please choose one

Eigen values for the system of the differential equations $X' = AX$ are evaluated for the

- ► Solution vector X
- ► Coefficient matrix A
- ► Differentiated solution vector X'
- ► Transpose of the Coefficient matrix A

Question No: 5 (Marks: 2) - Please choose one

Fundamental set of the solution vectors $X_1, X_2, \cdots, X_n$ for any system of the differential equations are obtained by

- ► Developing the singleton set $\{X\} = \{c_1X_1, c_2X_2, \ldots\ldots, c_nX_n\}$ of the linear combinitions of the solution vectors.
- ► Taking derivative of the each solution vector and forming the set $\{X_1', X_2', \cdots, X_n'\}$

▶ Taking Integral of the each solution vector and forming the set

$$\left\{\int X_1 dx, \int X_2 dx, \cdots, \int X_n dx\right\}$$

▶ Just verifying their linear independance and establishing the set

$$\{X_1, X_2, \cdots, X_n\}$$

.

Question No: 6 (Marks: 5)

For the family with parameter '**a**', of rectangular hyperbola $y = \frac{a}{x}$, find its corresponding orthogonal trajectorie and induce the parameter of the new family.

Question No: 7 (Marks: 5)

(a) Justify whether $D^2 + 9$ annihilates the function $f(x) = \sin 3x$ **(3)**

(b) Evaluate Wronskian of functions $Sin|x|$ and $|x|$ for $x < 0$ provided that $|x| = -x$ for $x < 0$ **(2)**

Question No: 8 (Marks: 8)

(a) Determine the interval of convergence of power series $\sum_{k=0}^{\infty} \frac{2^k}{k} x^k$
(5)

(b) If $\sum_{n=0}^{\infty} nc_n x^n$ and $\sum_{n=1}^{\infty} nc_{n-1} x^{n+1}$ are power series solutions for a differential equation then find their sum
by introducing a single summation symbol

Question No: 9 (Marks: 10)

(a) Develop a recurrence relation for the differential equation $y' - xy = 0$ by applying power series method. **(8)**

(b) Discuss shortly the linear independence of the power series solutions

$$y_1 = A\sqrt{x}\left[2 + \sum_{k=1}^{\infty} \frac{x^k}{(k+1)!5.8.11..........(3k+2)}\right]$$

and

$$y_2 = Bx\left[3 + \sum_{k=1}^{\infty} \frac{x^k}{k!1.4.7..........(3k-2)}\right]$$

Question No: 10 (Marks: 10)

(a) Determine the singular points of differential equation $y'' - \frac{1}{x}y' + \frac{1}{(x-1)^3}y = 0$ .Also classify each singular point as regular or irregular. **(5)**

(b) Using Rodrigues Formula; $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ to generate fourth Legendre's polynomial $P_3(x)$. **(5)**

Question No: 11 (Marks: 5)

Verify that the vector $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-5t}$ is a solution of the system ; $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dt} = 4x - 7y$.

	MIDTERM EXAMINATION SPRING 2007 MTH401 - DIFFERENTIAL EQUATIONS (Session - 4)	Marks: 40 Time: 90min

StudentID/LoginID: ______________________________

Student Name: ______________________________

Center Name/Code: ______________________________

Exam Date: Friday, May 18, 2007

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.

2. Do not ask any questions about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

c. Write all steps, missing steps may lead to deduction of marks.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

For Teacher's use only

Question	1	2	3	4	5	6	7	8	9		Total
Marks											

Question No: 1 (Marks: 1) - Please choose one

The differential equation

$(3x^2 y + 2)dx + (x^3 + y)dy = 0$ is

- Exact
- Linear
- Homogenous
- Separable

Question No: 2 (Marks: 1) - Please choose one

The assumed particular solution for the U.C(Undetermined Coefficient) differential equation $y' - y = x^2 e^{2x}$ is

- $y_p = c_1 e^{x^2} + c_2 x^2$
- $y_p = (Ax + B)e^{2x}$
- $y_p = (Ax^2 + Bx + c)e^{2x}$
- None of these.

Question No: 3 (Marks: 1) - Please choose one

The differential equation $x\frac{dy}{dx} + y = y^2 \ln x$ is an example of

- Separable
- Homogenous
- Exact
- None of these.

Question No: 4 (Marks: 1) - Please choose one

For the differential equation
$y' - 2xy = x$
Integrating factor is

- $-x^2$
- e^{x^2}

► e^{-x^2}

► x^2

Question No: 5 (Marks: 1) - Please choose one

$$\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$$

Identify the ordinary differential equation

► Homogenous

► Separable

► Exact

► None of these.

Question No: 6 (Marks: 5)

Solve the differential equation

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

Question No: 7 (Marks: 10)

Solve

$$(y Sec^2 x + Secx \tan x)\, dx + (\tan x + 2y)\, dy = 0$$

Question No: 8 (Marks: 10)

Find the equation of orthogonal trajectories of the curve

$$x^2 + y^2 = cx$$

Question No: 9 (Marks: 10)

Solve the differential equation by method of variations of parameters

$$\frac{d^2y}{dx^2} + y = \tan x \sec x$$

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